CS 112

Introduction to Data Structures

WEEK 08

Algorithm Analysis and Big-Oh

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This Week

1. Counting operations
2. From counts to classes
3. Big-Oh notation
4. Why constants drop out
5. The curves that matter **KEY**
6. Reading complexity off code
7. Best, worst, and amortized

How Long Does This Take?

```
v[i]                    // 3 operations, always

v.push_back(it)         // 5 operations if there's room
                        // ~3n + 5 if it must grow

list.push_front(it)     // 4 operations, always
```

> Counting exact operations is honest but useless for comparing algorithms, the number depends on the compiler, the machine, the mood of the cache. What we actually care about is **how the cost changes as *n* grows**.

Two Things We Don't Care About

Constant factors

```
3n + 5
100n + 2000
```

Both grow *proportionally to n*. Double n, double the work.

Lower-order terms

$n^2 + 50n + 700$

At n = 1000, the n^2 term is 20× everything else combined.

> So we throw both away and keep only the term that dominates. What's left is the **order of growth**.

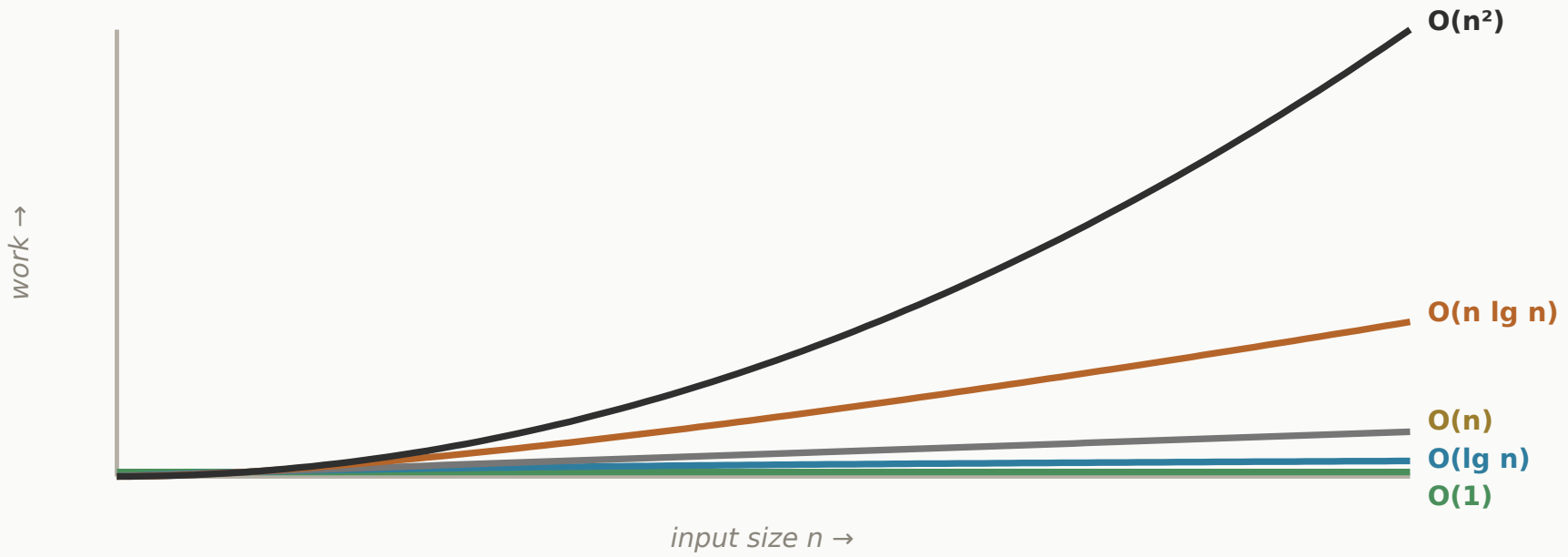
Big-Oh Notation

We write **O(f(n))**: "on the order of f of n".

Exact count	Big-Oh	Name
3	O(1)	constant
$\log_2 n + 4$	O(lg n)	logarithmic
$3n + 5$	O(n)	linear
$2n \lg n + n$	O(n lg n)	linearithmic
$n^2 + 50n + 700$	$O(n^2)$	quadratic
2^n	$O(2^n)$	exponential

> Big-Oh is an *upper bound on the shape of the growth*, not a promise about seconds.

The Curves That Matter



> Near the origin the differences look academic. That is exactly why they surprise people in production, the gap only opens up once the data gets big.

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An algorithm takes `4n + 200` steps. Its complexity is…

A. O(4n)

B. O(n)

C. O(n + 200)

D. O(200)

Reading It Off the Code

```
for (int i = 0; i < n; ++i) {
  total += a[i];
}
```

One loop over n → **O(n)**

```
for (int i = 0; i < n; ++i) {
  for (int j = 0; j < n; ++j) {
    if (a[i] == a[j]) ...
  }
}
```

Loop inside a loop → **O(n²)**

> Rule of thumb: loops *in sequence* add (keep the biggest), loops *nested* multiply. Halving the problem each pass gives you a lg n.

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What is the complexity of this loop?

```
for (int i = 1; i < n; i = i * 2) {
    cout << i << endl;
}
```

A. O(n)

B. O(n^2)

C. O(lg n)

D. O(n lg n)

What We've Built So Far

Operation	Dynamic array	Linked list
index `v[i]`	O(1)	O(n)
append	O(1) amortized	O(1)
prepend	O(n)	O(1)
insert / remove at position	O(n)	O(n)
search (unsorted)	O(n)	O(n)
traverse everything	O(n)	O(n)

> The same table you filled in by counting, now in the language everyone uses.

Why This Matters: Searching

One million items, and you need to find one.

Approach	Complexity	Comparisons
Linear search	O(n)	up to 1,000,000
Binary search (sorted)	O(lg n)	about **20**

> Fifty thousand times less work, from the same data, just organised differently. That is the entire argument for this course.

Demo

Timing O(n) against O(n²) on real data.

(same laptop, same data, wildly different patience)

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Algorithm A is $O(n^2)$, algorithm B is $O(n \lg n)$. Which should you use?

- **A.** Always B; it has the better complexity
- **B.** Always A, simpler code runs faster
- **C.** B for large n; for small n, A may well be faster
- **D.** Whichever is easier to write

Overloading `<<` for Your Container

```
// in List.h
ostream& operator<<(ostream &out, const List &list);

// in List.cpp
ostream& operator<<(ostream &out, const List &list) {
    list.writeTo(out);
    return out;
}
```

> Week 3's rule again: a free function, taking the stream on the left, returning the stream so it chains. Traversing the whole list to print it is O(n), as you'd expect.

Week 08 Recap

- Big-Oh describes **how work grows with n**, not seconds on a clock
- Drop constant factors and lower-order terms: keep the dominant one
- Nested loops multiply; halving the problem gives `lg n`
- $O(1) < O(\lg n) < O(n) < O(n \lg n) < O(n^2) < O(2^n)$
- At a million items the difference is a second versus an afternoon

Next: a09 →

This deck stands on earlier CS112 materials by
Joel Adams and **Victor Norman** · adapted and extended by **Eric Araújo**